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ONE-DIMENSIONAL NEUTRON TRANSPORT CALCULATIONS IN MULTILAYERED MEDIA BY ANALYTICAL DISCRETE ORDINATES METHOD

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ABSTRACT

In this work, a closed solution for a class of one-dimensional neutron transport problems in Cartesian geometry will be presented, considering linearly anisotropic scattering effects in heterogeneous media, using the Analytical Discrete Ordinates Method (ADO Method). In this context, the mathematical model will describe a steady-state phenomenon, with neutron sources located inside and on the boundaries of the domain of interest. In the process, the integro-differential transport equation is transformed into an ODE system by the S_N angular discretization, which homogeneous solution is obtained with a quadratic eigenvalues problem with reduced order. A particular solution in terms of constants is used. To validate the code, the method and provide benchmark results, test problems will be treated and results will be discussed.

1. INTRODUCTION

Nowadays, there are many engineering applications involving particles transport and radiation (Badruzzaman,1991; Sanchez,2012). Among these, the energy generation by nuclear sources has taken a prominent position, mainly for its zero emission of pollutants to the atmosphere. Considered as alternative energy sources, the nuclear power plants are characterized by producing large amounts of electricity from thermal energy generated by the controlled use of nuclear reactions. However, large volumes of water required for the cooling process and steam generation makes convenient to build these industrial facilities near to the rivers and the coast, raising concerns about the environmental impact.

In this sense, deep penetration problems, characterized by domains with several mean free paths of extension, have been object of particular interest because of their potential application in shielding calculation, which is directly related with personnel, equipment and environment protection (Oliveira, 2007).

Shielding problems have been dealt with in different ways by many researchers along the years. On deterministic methods field, Marchuk and Bel'skaya (1967) performed a shielding analysis applying spherical harmonics method associated with conjugated equations in a three-dimensional medium, while Veselov (1967) presented the spatial, angular and energy distribution of a three-dimensional neutrons transport in infinity medium using various approximations. In Giacomazzi (2000), the LTS_N method was used to compute the absorbed doses shielding in one-dimensional homogeneous and heterogeneous media, and the Adomian's decomposition method was presented by

Vargas et al. (2003) to solve linear and non-linear discrete-ordinates problems in one-dimensional geometries. On the other hand, using a probabilistic approach, Wagner and Haghighat (1998) used the Monte Carlo method associated with discrete ordinates adjoint functions for one-dimensional shielding calculations.

Therefore, the contribution of this work will be the development of a closed form solution for the discrete ordinates version of the integro-differential transport equation for a class of problems in one-dimensional Cartesian geometry, in homogeneous and heterogeneous media with linearly anisotropic scattering influence by the ADO method. The ADO method (Barichello and Siewert, 1999) has been successfully used for solving a wide range of one-dimensional RGD problems (Scherer, Prolo Filho and Barichello, 2009a; Scherer, Prolo Filho and Barichello, 2009b;) and neutron transport problems in different geometries (Barichello, Rodrigues and Siewert, 2002; Tres et al. 2014; Ferreira, Emmendorfer and Prolo Filho, 2015), offering accurate solutions in a concise manner through a simpler implementation code. Some advantages of this approach include the independence of iterative methods, the reduced order of the derivation-associated eigenvalues problem and the low computational cost. Besides that, the spatial variable is treated analytically, resulting in a more efficient formulation on the computational point of view.

In this way, in the next sections, the heterogeneous version of the one-dimensional discrete ordinates neutron transport equation is introduced. Then, the ADO method is applied, a reduced order eigenvalues problem is obtained, and the homogeneous solutions are explicitly defined. Due to the neutron source, particular solutions in terms of constants are shown, completing the general solution for the presented problems. In the end, computational aspects and numerical results are discussed.

2. MATHEMATICAL MODEL

According to Barros et al. (2010), the discrete ordinates version for a neutron transport equation in one-dimensional Cartesian geometry, applied to a layered heterogeneous medium with linearly anisotropic scattering, in steady-state regime, is written as

$$\mu_i \frac{d}{dx}\Psi_\alpha(x,\mu_i)+\sigma_{t,\alpha}\Psi_\alpha(x,\mu_i) = \frac{\sigma_{s0,\alpha}}{2}\sum_{k=1}^{N} w_k\,\Psi_\alpha(x,\mu_k) + \frac{3}{2}\sigma_{s1,\alpha}\mu_i \sum_{k=1}^{N} w_k\,\mu_k\Psi_\alpha(x,\mu_k)+Q_\alpha(x,\mu_i), \quad (1)$$

with i=1, ... ,N, being N associated to the number of discrete directions of the Gauss-Legendre quadrature set (Stroud and Secrest, 1966), and α=1, ... ,M, where M corresponds to the number of layers in which the domain is subdivided. Besides that, x [cm] and μ_i are, respectively, the spatial and directional variables wherein the angular fluxes Ψ_α [n/cm^2.s] are evaluated, w_k are the weights associated to the Gauss-Legendre points μ_k, Q_α [n/cm^3.s] represents a neutron source inside the layer α, $\sigma_{t,\alpha}$ [cm^{-1}], $\sigma_{s0,\alpha}$ [cm^{-1}] and $\sigma_{s1,\alpha}$ [cm^{-1}] are the total, isotropic and linearly anisotropic macroscopic cross sections for the layer α.

In order to apply the ADO method (Barichello and Siewert, 1999), the discrete directions are rearranged so that, for i=1, ... ,N/2, μ_i corresponds to the positive directions and $-\mu_i$ is relative to the negative directions. Consequently, Eq. (1) is subdivided in two, as

$$\mu_i \frac{d}{dx}\Psi_\alpha(x,\mu_i)+\sigma_{t,\alpha}\Psi_\alpha(x,\mu_i) = \frac{\sigma_{s0,\alpha}}{2}\sum_{k=1}^{N/2} w_k\left[\Psi_\alpha(x,\mu_k)+\Psi_\alpha(x,-\mu_k)\right] + \frac{3}{2}\sigma_{s1,\alpha}\mu_i \sum_{k=1}^{N/2} w_k\,\mu_k\left[\Psi_\alpha(x,\mu_k)-\Psi_\alpha(x,-\mu_k)\right]+Q_\alpha(x,\mu_i) \quad (2)$$

and

$$-\mu_i \frac{d}{dx}\Psi_\alpha(x,-\mu_i)+\sigma_{t,\alpha}\Psi_\alpha(x,-\mu_i) = \frac{\sigma_{s0,\alpha}}{2}\sum_{k=1}^{N/2} w_k\left[\Psi_\alpha(x,\mu_k)+\Psi_\alpha(x,-\mu_k)\right] - \frac{3}{2}\sigma_{s1,\alpha}\mu_i \sum_{k=1}^{N/2} w_k\,\mu_k\left[\Psi_\alpha(x,\mu_k)-\Psi_\alpha(x,-\mu_k)\right]+Q_\alpha(x,-\mu_i), \quad (3)$$

for i=1, ... ,N/2 and α=1, ... ,M.

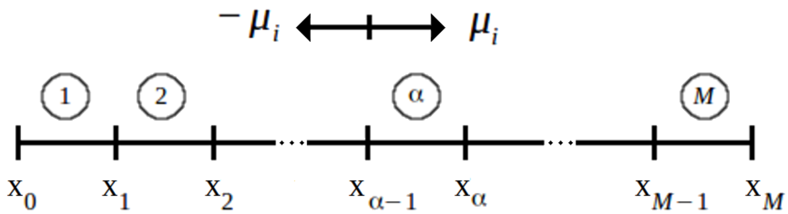

Figure 1. Domain of heterogeneous neutron transport problems.

On the cases presented here, the neutron sources can be located inside of each layer α and on the boundaries, so particular solutions will be needed.

3. THE ADO METHOD

Following some basic steps of the ADO method, a homogeneous solution for the transport problem described by Eqs. (2) and (3) is proposed in the form

$$\Psi_\alpha(x, \pm \mu_i)=\Phi_\alpha(\nu_\alpha, \pm\mu_i)e^{-x/\nu_\alpha}, \tag{4}$$

for i=1, ... ,N/2, α=1, ... ,M, where the separation constant ν_α is associated with the elementary solution $\Phi_\alpha(\nu_\alpha, \pm\mu_i)$.

This way, substituting Eq. (4) into Eqs. (2) and (3), the algebraic systems

$$-\frac{\mu_i}{\nu_\alpha}\Phi_\alpha(\nu_\alpha,\mu_i)+\sigma_{t,\alpha}\Phi_\alpha(\nu_\alpha,\mu_i) = \frac{\sigma_{s0,\alpha}}{2}\sum_{k=1}^{N/2} w_k\left[\Phi_\alpha(\nu_\alpha,\mu_k)+\Phi_\alpha(\nu_\alpha,-\mu_k)\right] + \frac{3}{2}\sigma_{s1,\alpha}\mu_i\sum_{k=1}^{N/2} w_k\,\mu_k\left[\Phi_\alpha(\nu_\alpha,\mu_k) - \Phi_\alpha(\nu_\alpha,-\mu_k)\right] \tag{5}$$

and

$$\frac{\mu_i}{\nu_\alpha}\Phi_\alpha(\nu_\alpha,-\mu_i)+\sigma_{t,\alpha}\Phi_\alpha(\nu_\alpha,-\mu_i) = \frac{\sigma_{s0,\alpha}}{2}\sum_{k=1}^{N/2} w_k\left[\Phi_\alpha(\nu_\alpha,\mu_k)+\Phi_\alpha(\nu_\alpha,-\mu_k)\right] - \frac{3}{2}\sigma_{s1,\alpha}\mu_i\sum_{k=1}^{N/2} w_k\,\mu_k\left[\Phi_\alpha(\nu_\alpha,\mu_k) - \Phi_\alpha(\nu_\alpha,-\mu_k)\right], \tag{6}$$

for i=1, ... ,N/2 and α=1, ... ,M are obtained.

Now, two auxiliary functions are defined as

$$U_\alpha(\nu_\alpha,\mu_i)=\Phi_\alpha(\nu_\alpha,\mu_i)+\Phi_\alpha(\nu_\alpha,-\mu_i), \tag{7}$$

$$V_\alpha(\nu_\alpha,\mu_i)=\Phi_\alpha(\nu_\alpha,\mu_i) - \Phi_\alpha(\nu_\alpha,-\mu_i), \tag{8}$$

such that, if Eqs. (5) and (6) are added, the expression

$$V_\alpha(\nu_\alpha,\mu_i) = \frac{\nu_\alpha}{\mu_i}\left[\sigma_{t,\alpha}U_\alpha(\nu_\alpha,\mu_i) - \sigma_{s0,\alpha}\sum_{k=1}^{N/2} w_k\,U_\alpha(\nu_\alpha,\mu_k)\right] \tag{9}$$

is obtained.

Now, subtracting Eq. (6) from Eq. (5), another relation between $U_\alpha(\nu_\alpha,\mu_i)$ and $V_\alpha(\nu_\alpha,\mu_i)$ is obtained, and it is given by

$$-\frac{\mu_i}{\nu_\alpha}U_\alpha(\nu_\alpha,\mu_i)+\sigma_{t,\alpha}V_\alpha(\nu_\alpha,\mu_i) = 3\sigma_{s1,\alpha}\mu_i\sum_{k=1}^{N/2} w_k\,\mu_k V_\alpha(\nu_\alpha,\mu_k). \tag{10}$$

From Eqs. (9) and (10), after some algebraic manipulations, an eigenvalue problem in terms of $U_\alpha(\nu_\alpha,\mu_i)$ is obtained, in the form

$$\frac{1}{\nu_\alpha^2}U_\alpha(\nu_\alpha,\mu_i) = \frac{\sigma_{t,\alpha}^2}{\mu_i^2}U_\alpha(\nu_\alpha,\mu_i) - \sum_{k=1}^{N/2} w_k\left[\frac{\sigma_{t,\alpha}\sigma_{s0,\alpha}}{\mu_i^2} + 3\sigma_{t,\alpha}\sigma_{s1,\alpha} - 3\sigma_{s0,\alpha}\sigma_{s1,\alpha}\left(\sum_{j=1}^{N/2} w_j\right)\right]U_\alpha(\nu_\alpha,\mu_k), \tag{11}$$

for i=1, ... ,N/2 and α=1, ... ,M. The matrix representation of Eq. (11) is given by

$$[D_\alpha - A_\alpha]\overrightarrow{U_\alpha}=\lambda_\alpha\overrightarrow{U_\alpha}, \tag{12}$$

where $\overrightarrow{U_\alpha}$ is a vector with components $U_\alpha(\nu_\alpha,\mu_i)$, and

$$\lambda_\alpha = \frac{1}{\nu_\alpha^2}. \tag{13}$$

The $N/2 \times N/2$ matrices in Eq. (12) are such that

$$D_\alpha = \text{diag}\left[\frac{\sigma_{t,\alpha}^2}{\mu_1^2}, \frac{\sigma_{t,\alpha}^2}{\mu_2^2}, \dots, \frac{\sigma_{t,\alpha}^2}{\mu_{N/2}^2}\right] \tag{14}$$

and

$$A_\alpha(i,k)=w_k\left[\frac{\sigma_{t,\alpha}\sigma_{s0,\alpha}}{\mu_i^2} + 3\sigma_{t,\alpha}\sigma_{s1,\alpha} - 3\sigma_{s0,\alpha}\sigma_{s1,\alpha}\left(\sum_{j=1}^{N/2} w_j\right)\right], \tag{15}$$

for i,k=1, ... ,N/2 and α=1, ... ,M.

With the eigenvalue problem solved, the values of $\lambda_{\alpha,j}$ for j=1, ... ,N/2 are obtained, such that the values of the separation constants $\nu_{\alpha,j}$ are found by Eq. (13) and, from Eqs. (7) and (8), the elementary solution can be written as

$$\Phi_\alpha(\nu_{\alpha,j},\mu_i) = \frac{1}{2}\left[U_\alpha(\nu_{\alpha,j},\mu_i)+V_\alpha(\nu_{\alpha,j},\mu_i)\right] \tag{16}$$

and

$$\Phi_\alpha(\nu_{\alpha,j},-\mu_i) = \frac{1}{2}\left[U_\alpha(\nu_{\alpha,j},\mu_i) - V_\alpha(\nu_{\alpha,j},\mu_i)\right], \tag{17}$$

for i,j=1, ... ,N/2 and α=1, ... ,M.

Since the separation constants occur in pairs, $\{\pm\nu_{\alpha,j}\}$, with real values, and using the symmetry properties of the elementary solutions

$$\Phi_\alpha(\nu_{\alpha,j},\mu_i)=\Phi_\alpha(-\nu_{\alpha,j},-\mu_i), \tag{18}$$

$$\Phi_\alpha(\nu_{\alpha,j},-\mu_i)=\Phi_\alpha(-\nu_{\alpha,j},\mu_i), \tag{19}$$

the homogeneous solutions for Eqs. (2) and (3), in an explicit form, are given by

$$\Psi_\alpha^h(x,\mu_i) = \sum_{j=1}^{N/2} A_{\alpha,j}\,\Phi_\alpha(\nu_{\alpha,j},\mu_i)e^{-(x-x_{\alpha-1})/\nu_{\alpha,j}} + A_{\alpha,j+N/2}\Phi_\alpha(-\nu_{\alpha,j},\mu_i)e^{-(x_\alpha-x)/\nu_{\alpha,j}} \tag{20}$$

and

$$\Psi_\alpha^h(x,-\mu_i) = \sum_{j=1}^{N/2} A_{\alpha,j}\,\Phi_\alpha(\nu_{\alpha,j},-\mu_i)e^{-(x-x_{\alpha-1})/\nu_{\alpha,j}} + A_{\alpha,j+N/2}\Phi_\alpha(-\nu_{\alpha,j},-\mu_i)e^{-(x_\alpha-x)/\nu_{\alpha,j}}, \tag{21}$$

for i=1, ... ,N/2 and each region α=1, ... ,M. Here, the arbitrary constants $A_{\alpha,j}$ are to be determined, and they depend on the boundary conditions and particular solutions. Despite all boundary conditions used here are prescribed, the ADO method has proven also effective in dealing with more general boundary conditions.

It is important to observe that, in this formulation, from a set of N discrete ordinates equations, an eigenvalue problem of order N/2 was derived, which means a relevant gain in comparison with other similar discrete ordinates approaches, where characteristic equations or eigensystems of order N are obtained, for the same quadrature scheme (Vilhena and Barichello, 1991; Nunes and Barros, 2009). Furthermore, the expressions for the homogeneous solutions, in terms of spatial variable, are analytical, contributing to the low computational cost and high accuracy of the method.

4. PARTICULAR SOLUTION

Since the problem formulated by Eq. (1) has a non-homogeneous source term, particular solutions have to be defined. For that, a simpler particular solution can be considered for each layer α and each direction in terms of constants. So, for i=1, ... ,N/2,

$$\Psi_\alpha^p(x,\mu_i) = B_{\alpha,i} \tag{22}$$

and

$$\Psi_\alpha^p(x,-\mu_i) = C_{\alpha,i} \tag{23}$$

are taken such that, substituting them into Eqs. (2) and (3), a coupled NxN linear system is obtained, as follows

$$[P_\alpha - R_\alpha]\overrightarrow{O_\alpha} = \overrightarrow{S_\alpha}, \tag{24}$$

where

$$P_\alpha = \mathrm{diag}\left[\sigma_{t,\alpha}, \dots, \sigma_{t,\alpha}\right], \tag{25}$$

$$R_\alpha(i,k) = \begin{bmatrix} \left[\frac{\sigma_{s0,\alpha}}{2} w_k + \frac{3}{2}\sigma_{s1,\alpha}\mu_i\mu_k w_k\right] & \left[\frac{\sigma_{s0,\alpha}}{2} w_k - \frac{3}{2}\sigma_{s1,\alpha}\mu_i\mu_k w_k\right] \\ \left[\frac{\sigma_{s0,\alpha}}{2} w_k - \frac{3}{2}\sigma_{s1,\alpha}\mu_i\mu_k w_k\right] & \left[\frac{\sigma_{s0,\alpha}}{2} w_k + \frac{3}{2}\sigma_{s1,\alpha}\mu_i\mu_k w_k\right] \end{bmatrix}, \tag{26}$$

$$\overrightarrow{O_\alpha} = \begin{bmatrix} [B_{\alpha,i}] \\ [C_{\alpha,i}] \end{bmatrix} \tag{27}$$

and

$$\overrightarrow{S_\alpha} = \begin{bmatrix} [Q_\alpha(x,\mu_i)] \\ [Q_\alpha(x,-\mu_i)] \end{bmatrix}, \tag{28}$$

for i,k=1, … ,N/2 and α=1, … M.

Solved the linear system, the particular solutions are obtained and the general solutions will be given by

$$\Psi_\alpha(x,\pm\mu_i) = \Psi_\alpha^h(x,\pm\mu_i) + \Psi_\alpha^p(x,\pm\mu_i) \tag{29}$$

for i=1, … ,N/2 and α=1, … M.

5. COUPLING SYSTEM

In order to explicitly define the solutions for the class of problems proposed here, boundary and interface conditions are needed. Then,

$$\Psi_1(x_0,\mu_i) = F_i, \tag{30}$$

$$\Psi_M(x_M,-\mu_i) = G_i, \tag{31}$$

are used for i=1, … ,N/2, being F_i and G_i the constant incident fluxes at the boundaries and, to ensure the uniformity of the fluxes between the neighboring regions, the interface condition is given by

$$\Psi_\alpha(x_\alpha,\pm\mu_i) = \Psi_{\alpha+1}(x_\alpha,\pm\mu_i), \tag{32}$$

for i=1, … ,N/2 and α=1, … ,$M-1$.

The Eqs. (30)-(32) lead to a $MN \times MN$ system which solution provides the value of all coefficients $A_{\alpha,j}$ and, consequently, makes Eq. (29) become completely established. After that, relevant quantities can be computed, such as the *Scalar Flux*.

6. NUMERICAL RESULTS AND COMPUTATIONAL ASPECTS

For the approach presented here, two test cases were considered where, in one of them, it was possible to compare the ADO method with other numerical methods for validation. In order to generate the results profiles, the *Scalar Flux* was chosen as quantity of interest, defined for each layer α by

$$\phi_\alpha(x) = \frac{1}{2}\sum_{k=1}^{N/2} w_k\,[\Psi_\alpha(x,\mu_k) + \Psi_\alpha(x,-\mu_k)], \tag{33}$$

corresponding to the average angular flux in terms of the directional variable.

In Nunes and Barros (2009), *Problem 1* was solved following the parameters presented on Tab. 1, considering a 40cm domain width, with vacuum boundary condition on $x = 40\text{cm}$ and unitary incident flux on $x = 0\text{cm}$. On the related reference, three different numerical methods are used: *Diamond difference method* (DD), *Step method* and *CN method*. All of them subdivide the domain into cells, where average angular fluxes are established. The numeric aspect consists in how the relationship between neighboring cells is, and an iterative process is used to compute it.

Table 1. Parameters used for Problem 1.

	Region 1	Region 2	Region 3
x	$0.0 \leq x \leq 15.0$	$15.0 \leq x \leq 30.0$	$30.0 \leq x \leq 40.0$
$\sigma_{t,\alpha}$	1.00	1.00	1.00
$\sigma_{s0,\alpha}$	0.97	0.95	0.99
$\sigma_{s1,\alpha}$	0.00	0.00	0.00
$Q_\alpha(x, \pm\mu_i)$	0.00	0.00	1.00

On the other hand, with the ADO method, it is possible to obtain explicit expressions for the solutions (analytical in terms of the spatial variable), making it possible to compute the scalar fluxes values in any spatial position of interest without using iterative methods or interpolation process.

In terms of agreement, the ADO method becomes closer to DD method (Tab. 2), getting three to five significant digits of concordance. The resulting shape presented on Fig. 2 agrees with the proposed parameters, the phenomenon and the values of Tab. 2. On the convergence analysis (Tab. 3), two to five significant fixed digits are shown, still being possible to obtain more accurate profiles just by increasing the value of N.

Table 2. Problem 1 — Validation of Scalar Fluxes profiles. Comparison among different methods using S_8 quadrature scheme.

x	DD	STEP	CN	ADO
0.00	0.852638	0.851075	0.850716	0.852637
20.00	0.481766	0.512658	0.504515	0.481821
40.00	7.090416	7.126999	7.245023	7.090392

Table 3. Problem 1 — Convergence analysis of Scalar Fluxes profiles computed by ADO method (this work).

x	S_2	S_4	S_6	S_8
0.00	0.852605	0.852638	0.852638	0.852637
20.00	0.459243	0.481301	0.481703	0.481821
40.00	7.045806	7.086130	7.089433	7.090392

Problem 2 approaches a five regions transport problem, following the parameters defined on Tab. 4, considering a 50cm domain width, with vacuum boundary conditions on $x = 0\text{cm}$ and $x = 50\text{cm}$.

It is important to highlight (about the ADO method) that it wasn't necessary to deal with ill-conditioned systems and, despite splitting the domain in layers, in every and each of the parts, explicit analytical solutions (in terms of the spatial variable) were obtained. Besides that, all the parameters that arise during the process are real, so the use of complex variable techniques is not necessary. Furthermore, the convergence for *Problem 2* can be noted on Tab. 5, where up to three digits can be fixed by increasing the value of N.

In order to complete the study about the behavior of the Scalar Flux in a layered heterogeneous problem, the Fig. 3 shows that while layers with bigger isotropic scattering reach higher values, regions with bigger anisotropic scattering suffer a kind of flattening effect, that means the isotropic scattering contribute more to the height of the profiles, even with the source terms contribution.

Table 4. Parameters used for Problem 2.

	Region 1	Region 2	Region 3	Region 4	Region 5
x	$0.0 \le x \le 10.0$	$10.0 \le x \le 20.0$	$20.0 \le x \le 30.0$	$30.0 \le x \le 40.0$	$40.0 \le x \le 50.0$
$\sigma_{t,\alpha}$	1.00	1.00	1.00	1.00	1.00
$\sigma_{s0,\alpha}$	0.98	0.96	0.94	0.92	0.90
$\sigma_{s1,\alpha}$	0.90	0.92	0.94	0.96	0.98
$Q_\alpha(x, \pm\mu_i)$	0.00	1.00	0.00	1.00	0.00

Table 5. Problem 2 — Convergence analysis of Scalar Fluxes profiles computed by ADO method (this work).

x	S_2	S_4	S_6	S_8
0.00	1.866138	1.760767	1.752773	1.750319
10.00	10.939594	11.044818	11.050256	11.051874
20.00	9.789710	9.812068	9.813365	9.813713
30.00	7.146850	7.127936	7.128151	7.128202
40.00	5.038048	5.074692	5.075679	5.075948
50.00	0.721845	0.674190	0.668338	0.666519

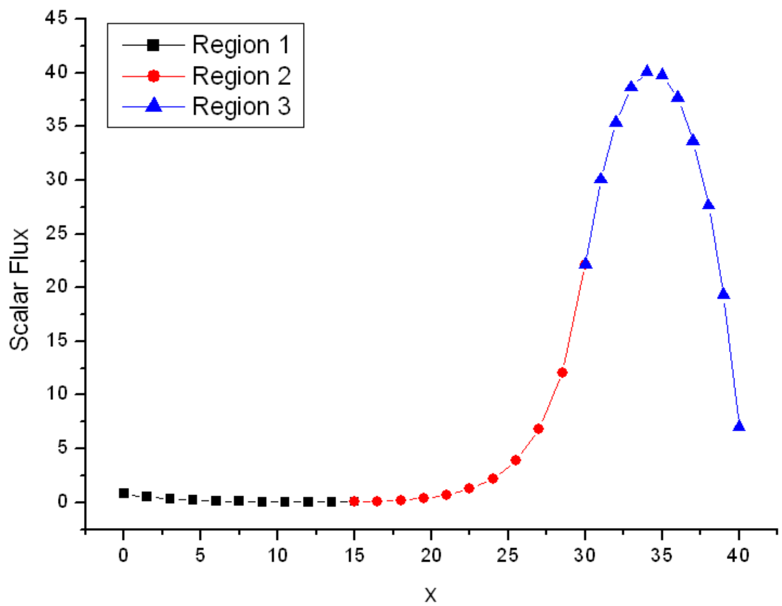

Figure 2. Scalar Fluxes profiles for the *Problem 1* computed by ADO method (this work) using S_8 quadrature scheme.

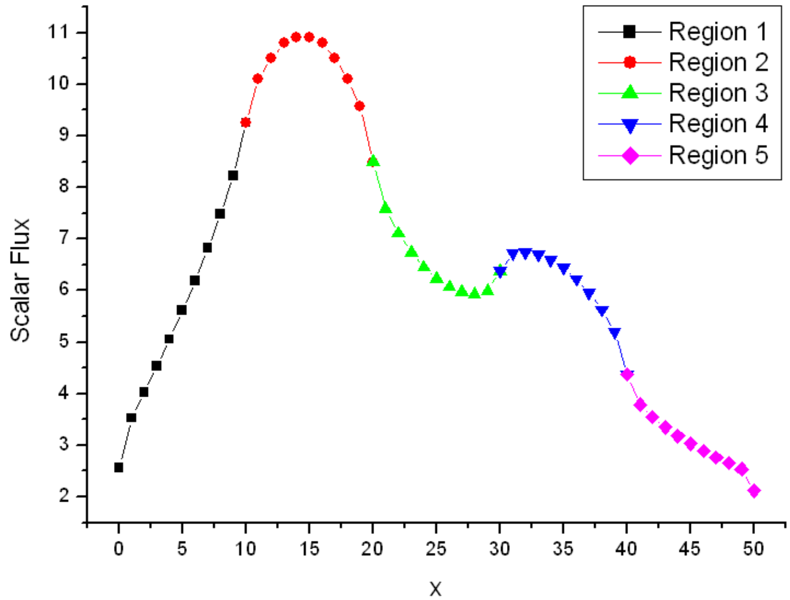

Figure 3. Scalar Fluxes profiles for the *Problem 2* computed by ADO method (this work) using S_8 quadrature scheme.

7. CONCLUSIONS

The present work shows the viability and performance of the ADO method in the solution of some classes of neutron transport problems, in one-dimensional Cartesian geometry, where it was possible to compare some results with the available literature and provide some benchmark profiles.

In the tables presented in this study, convergence of the results is noted when changing the Gauss-Legendre quadrature order. The increasing of the number of quadrature points leads to a better representation of the integral term, despite also increasing the number of discrete directions. In general, the profiles present a good agreement with the literature, approximately three to five significant digits. Also, it was observed that the anisotropy factor of the medium causes some shape effects on the Scalar Flux.

Here, some good features of the ADO method can be highlighted: hence it doesn't use computational spatial grid to evaluate the angular fluxes, the calculations can be made without iterative schemes, making the computational effort relatively low and spending less than 2 seconds (in a 3.10 GHz Intel Core I5 processor with 8GB of RAM) for each profile. Part of this performance was also due to the reduced order eigenvalue systems and the explicit form of the solutions, that are analytical in terms of the spatial variable. The code, which implementation is simple, was developed making use of the free software Octave 4.0, accepting arbitrary quadrature schemes, and working with any quadrature order.

Thus, the objectives proposed in this work can be considered achieved, as it was possible to provide closed form solutions to the proposed problems in a concise and accurate way, showing profiles with compatible physical behavior in terms of parameters, proposal of solutions and boundary conditions used.

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9. REFERENCES

Badruzzaman, A., 1991. “Computational methods in nuclear geophysics”. Progress in Nuclear Energy, Vol. 25, pp. 265-290.

Barichello, L.B. and Siewert, C.E., 1999. “A discrete-ordinates solution for a non-grey model with complete frequency redistribution”, JQSRT, Vol. 62, pp. 645-675.

Barichello, L.B., Rodrigues, P. and Siewert, C.E., 2002. “An analytical discrete-ordinates solution for dual-mode heat transfer in a cylinder”, JQSRT, Vol. 73, pp. 583-602.

Barros, R.C., Alves Filho, H., Plat, G.M., Oliveira, F.B.S. and Militão D.S., 2010. "Analytical reconstruction scheme for the coarse-mesh solution generated by the spectral nodal method for neutral particle discrete ordinates transport model in slab geometry". Annals of Nuclear Energy, Vol. 37, pp. 1461-1466.

Ferreira, C.E.S., Emmendorfer, L.R., and Prolo Filho, J.F., 2015. "Formulação nodal aplicada à problemas de transporte bidimensional em geometria cartesiana", Scientia Plena, Vol. 11, pp. 1-10.

Giacomazzi, E.T.P., 2000. "Cálculo de dose absorvida em blindagens múltiplas, devido a nêutrons monoenergéticos, usando o método LTSN", Master dissetation, UFRGS, Porto Alegre.

Marchuk, G.I. and Bel'skaya, Z.N., 1967. "Application of conjugate equations to the calculation of radiation shielding", NASA Technical Reports, NASA TT F-411.

Nunes, C.E.A., Barros, R.C., 2009. "Aplicativo computacional para cálculo de blindagem com modelo de transporte S_N unidimensional e monoenergético", In Proceedings of INAC 2009 International Nuclear Atlantic Conference, Rio de Janeiro, Brazil.

Oliveira, F.B.S., 2007. "Problema inverso de reconstrução analítica aproximada da solução da equação de transporte de partículas neutras monoenergéticas em geometria unidimensional cartesiana com espalhamento isotrópico". Ph.D. thesis, UERJ, Rio de Janeiro, Brazil.

Sanchez, R., 2012. "Prospects in deterministic three-dimensional whole-core transport calculations", Nuclear Engineering and Technology, Vol. 44, pp.113-150.

Scherer, C.S., Prolo Filho, J.F. and Barichello, L.B., 2009a. "An analytical approach to the unified solution of kinetic equations in rarefied gas dynamics. I. Flow problems", Z. Angew. Math. Phys., Vol. 60, pp. 70-115.

Scherer, C.S., Prolo Filho, J.F. and Barichello, L.B., 2009b. "An analytical approach to the unified solution of kinetic equations in rarefied gas dynamics. II. Heat transfer problems", Z. Angew. Math. Phys., Vol. 60, pp. 651-687.

Stroud, A.H., Secrest, D., 1966. "Gaussian quadrature formulas", Prentice-Hall Inc, New Jersey, USA.

Tres A., Picoloto, C.B, Prolo Filho, J.F., Da Cunha, R.D. and Barichello, L.B., 2014."Explicit formulation of a nodal transport method for discrete ordinates calculations in two-dimensional fixed-source problems", Kerntechnik, Vol. 79, pp. 155-162.

Veselov, V.M., 1967. "Investigation of the accuracy of various approximations in the problem of three-dimensional energy and angular distribution of neutrons", NASA Technical Reports, NASA TT F-411.

Vargas, R.M.F., Cardona, A.V., Vilhena, M.T. and Barros, R.C., 2003. "On the decomposition method applied to linear and non-linear discrete ordinates problems in slab geometry", Progress in nuclear energy, Vol. 42, No. 4, pp. 439-456.

Vilhena, M.T., Barichello, L.B., 1991. "A new analytical approach to solve the neutron transport equation", Kerntechnik, Vol. 56 pp. 334-336.

Wagner, C.J. and Haghighat A., 1998. "Automated variance reduction of Monte Carlo shielding calculations using the discrete ordinates adjoint function". Nuclear Science And Engineering, Vol. 128, pp. 186–208.

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